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Novel global optimization approach for the simultaneous design of heat exchangers in heat exchanger networks constrained by pressure drop

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ABSTRACT

We address the simultaneous globally optimal basic design of shell-and-tube heat exchangers interconnected in a HEN, where the total pressure drop of the streams that pass through several exchangers is limited to a maximum. We explore the optimal distribution of the available pressure drop in each stream among different heat exchangers, instead of the traditional procedure based on individual values of available pressure drop. The approach is useful in the context of HEN synthesis based on network structure enumeration when the design of the equipment is embedded in the algorithm. The objective is to obtain the optimal geometry of each exchanger, such that the total area or investment cost is minimized subject to the collective pressure drop constraints. The design variables for the shell-and-tube heat exchangers include all typical geometric dimensions. The novel global optimization approach builds an integer linear optimization model (ILOM), which is solved using an integer linear programming method (ILP). The ILOM involves only feasible heat exchanger candidates, obtained using Complete Set Trimming. The procedure was applied to two test cases involving 8 and 23 heat exchangers, respectively, each exchanger having 12,852,000 initial candidate geometries. The computational times for this task are below one hour for the two studied cases when implemented in MATLAB. Results for optimizing using the total annualized cost (CAPEX+OPEX) as an objective function and without pressure drop constraints are also presented for comparison.

1. Introduction

The synthesis of Heat Exchanger Networks (HENs) continues to be an important problem especially because decarbonization is already a must. Usually, this synthesis is conducted without major consideration for the design of the different exchangers and the corresponding pressure drop. However, the pressure drop is of importance because there are limitations on the pumps that can be used and there is an associated pumping cost that is not negligible. If the pressure drop of streams is to be considered a part of the constraints in HEN synthesis such pressure drop has to be obtained by considering the basic design of the heat exchangers, that is, incorporate the geometry of the exchangers. Several approaches exist for the combined synthesis of the HEN structure and the design of the exchangers using MINLP procedures, Metaheuristics, and Enumeration (see Oliva et al. (2024) for a review), but these do not consider the optimization of the pressure drop distribution. In particular, Enumeration approaches (Oliva et al., 2024), build different networks and obtain the optimal geometries of the exchangers together with the energy consumption. Such an approach is therefore amenable to be altered by considering the pressure drop in the optimization of each structure. In other less elaborated scenarios, a designer has already determined the process flowsheet and the thermal tasks of each exchanger and has also identified maximum pressure drops of specific streams, be it because the initial pressure is set by upstream equipment and the lower pressure of the final destination of the stream cannot be smaller than a certain value, or because there are limitations in the selection of the pumps to be used for that stream. This article presents a novel procedure to obtain the design of the exchangers for a fixed network and well-defined thermal tasks, such that the distribution of the available pressure drop of each stream among the different heat exchangers is optimized, instead of using a limiting fixed value for each

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Nomenclature

A_j	Heat exchanger area
$A_{j,i}$	Heat exchanger area of a solution candidate
$\widehat{a}_{exc,j}$	Excess area
$\widehat{b}$	Parameter of the heat exchanger capital cost evaluation
Bc	Baffle cut ratio
$CCOST_j$	Heat exchanger capital cost
$\widehat{C}_{EF}$	Parameter of the heat exchanger capital cost evaluation
$\widehat{C}_{EV}$	Parameter of the heat exchanger capital cost evaluation
Cp	Stream heat capacity
Ds	Shell diameter
dte	Outer tube diameter
F	LMTD correction factor
FS_j	Feasible set
HEN	Heat exchanger network
IS_j	Initial set
i	Solution candidate index
j	Heat exchanger index
k	Stream index
k	Thermal conductivity
K_j	Sum of cardinalities of the sets RS_j
L	Tube length
lay	Tube layout
lbc	Baffle spacing
$LMTD$	Logarithmic mean temperature difference
Nb	Number of baffles
Npt	Number of tube passes
P_k	Set of heat exchangers associated with a given stream
PS_j	Primordial set
$\widehat{Q}_j$	Heat exchanger heat load
Npt	Number of tube passes
rp	Tube pitch ratio
Res	Shell-side Reynolds number
$\widehat{Resmax}$	Upper bound of the shell-side Reynolds number
$\widehat{Resmin}$	Lower bound of the shell-side Reynolds number
Ret	Tube-side Reynolds number
$\widehat{Retmax}$	Upper bound of the tube-side Reynolds number
$\widehat{Retmin}$	Lower bound of the tube-side Reynolds number
rp	Tube pich ratio
TAC	Total annualized cost
$TCCOST$	Total capital cost of all heat exchangers
U	Overall heat transfer coefficient
vs	Shell side velocity
$\widehat{vsmax}$	Upper bound of shell side velocity
$\widehat{vsmin}$	Lower bound of shell side velocity
vt	Tube side velocity
$\widehat{vtmax}$	Upper bound of tube side velocity
$\widehat{vtmin}$	Lower bound of tube side velocity
$\underline{x}_j$	Vector of design variables of a heat exchanger
$\underline{x}_{j,i}$	Vector of design variables of a solution candidate
$\underline{y}_{j,i}$	Binary variable associated with a solution candidate

Greek Letters

$\Delta P_{j,k}$	Heat exchanger pressure drop
$\widehat{\Delta Plim}_k$	Stream pressure drop limit
ρ	Density
μ	Viscosity

heat exchanger, as it is usually employed. The potential gain to explore the distribution of available pressure drop among the different heat exchangers associated with the same stream was mentioned by several authors in the technical literature (Steinmeyer, 1996; Mukherjee, 1998).

The problem of designing individual heat exchangers is a very old problem. Traditionally, this problem was handled using successive trial geometries, followed by a verification if the heat transfer task can be accomplished and the pressure drop is within limits. This process, which is popular in several textbooks (Saunders, 1988; Kakaç and Liu, 2002; Serth, 2007; Cao, 2010) may render areas larger than the minimum possible (Costa and Bagajewicz, 2019), additionally, the quality of its solution depends on the skill of the engineer responsible for the task. Later, several design approaches were proposed using optimization. These attempts range from the use of metaheuristics (Chaudhuri et al., 1997; Sanaye et al., 2010; Mariani et al., 2012; Lahiri et al., 2014; Asadi et al., 2014; Mohanty, 2016; Iyer et al., 2019; Hajabdollahi et al., 2017; Mirzaei et al., 2017) to the use of mathematical programming (Jegede and Polley, 1992; Reppich and Kohoutek, 1994; Mizutani et al., 2003; Ponce-Ortega et al., 2006; Ravagnani and Caballero, 2007). Most of these attempts focused on minimizing area, which is used as a proxy for investment cost. Metaheuristics are based on mixed integer nonlinear models, and, by construction, cannot guarantee global optimality and suffer from other limitations, like the need for algorithm control parameter tuning, or sometimes the need for multiple repeated runs, etc. Mathematical programming is also based on mixed integer nonlinear models. Because of the non-convexities in the model, several mathematical mixed-integer nonlinear programming (MINLP) approaches render local optima, not being able to guarantee global optimality. In addition, when global solvers are used, they sometimes fail or take large computational time. Later, some reformulation techniques were proposed that render the problem linear (Gonçalves et al., 2017a,b, 2019) and consequently capable of providing globally optimal solutions. Finally, the use of Complete Set Trimming (Lemos et al., 2020) showed the same capabilities, but visibly improved computational time. Complete Set Trimming was first introduced by Costa and Bagajewicz (2019) by generalizing an idea developed informally by Gut and Pinto (2004) for the specific case of plate exchangers. This method is based on a different representation of the search space (candidate solutions vs. variables) and the sequential application of all the constraints to the original set, trimming it at each application and rendering a final set of feasible solutions, from which the optimal solution is determined by sorting. Later, heat transfer equipment that handles fluids with properties varying with temperature was solved using Partial Set Trimming followed by Smart Enumeration. In turn, Smart Enumeration is based on listing all candidates that survived the Partial Set Trimming in order of their lower bound and evaluating each candidate rigorously until the lower bound is larger than the incumbent upper bound (Costa and Bagajewicz, 2019; Nahes et al., 2022).

The definition of a limit on the total pressure drop for a stream passing through a set of heat exchangers forces to seek the optimal design of all of the units simultaneously. This problem was not fully addressed in the literature before. For example, Polley et al. (1990) briefly addressed the issue of the retrofit of heat exchanger networks. This analysis is particularly important when addressing a more complex problem of synthesizing heat exchanger networks (HENs), which is not considered in this article. This field of knowledge has produced 4000+ articles since the 1970s about this subject (Li et al., 2022a), but only a limited number of papers have started to consider the simultaneous synthesis of the network together with the design of the heat exchangers, i.e. the additional burden of having to determine the optimal design of the set of heat exchangers present in the HEN (Liporace et al., 1999; Ponce-Ortega et al.; 2007; Ravagnani and Silva, 2012; Xiao et al., 2019;

Karimi et al., 2020; Kazi et al., 2020,2021; Cotrim et al., 2021; Farzin et al., 2021; Oliva et al., 2024).

The limitations on pressure drop add another layer of complexity. Costa and Queiroz (2004), raised the issue of minimizing the cost of two exchangers in series where a cold stream matches with two hot streams in each exchanger, subject to a limitation of the total pressure drop for the stream in series. They show that taking the total available pressure drop for the cold stream and assigning equal available pressure drops to each exchanger is more expensive (+5.8 %) than allowing the cold stream to be subject only to a total pressure drop and letting a nonlinear optimization problem pick the pressure drops in each exchanger.

Considering that enumeration techniques have been developed that allow a rigorous decomposition procedure to identify a guaranteed globally optimal solution to this simultaneous network synthesis/ exchanger design, the technique presented in this article can be easily added to such a decomposition procedure without having to make any approximations.

The present paper proposes a new approach to the design of multiple interconnected heat exchangers. We first use Complete Set Trimming to obtain a set of feasible solutions for each shell-and-tube heat exchanger (Lemos et al., 2020). Then, these sets are reduced using information on the area and pressure drop of each exchanger. Finally, the global optimal solution is found involving the identification of the minimum cost combination of feasible designs that satisfy the system's pressure drop constraints by solving an integer linear optimization model (ILOM), which we solve using integer linear programming (ILP).

The remainder of the article is organized as follows. Section 2 presents the problem definition. Section 3 discusses how the Set Trimming procedure identifies the feasible set of heat exchanger candidates. Section 4 presents the integer linear optimization problem that attains the optimal set of solution candidates for each heat exchanger in the network. Section 5 summarizes the complete global optimization procedure. Finally, we finish presenting results and conclusions in Sections 6 and 7.

2. Problem definition

Consider a heat exchanger network with a defined set of heat transfer tasks and pressure drop limitations for all or some selected streams. Each thermal task corresponds to a heat exchanger j that must be designed. It is desired to find the geometry for each exchanger, such that the minimum cost is obtained when limits on the pressure drop of the streams through the network are imposed. The minimum cost considered here is the investment cost (or its proxy, the area).

The required area and pressure drop in each exchanger is a function of the discrete independent values of the design variables, represented by $\underline{x}_j$. For shell-and-tube heat exchangers, which we use in this article, they are: tube and shell diameters, tube length, number of baffles, number of tube-side passes, etc. These are subject to a set of constraints expressed generically as a vector, as follows:

$$\underline{g}_j\left(\underline{x}_j\right) \leq 0 \tag{1}$$

These constraints are described by Lemos et al. (2020) and they include, aspect ratios, velocity as well as Reynolds lower and upper bounds, etc. In particular, they include the requirement that the area is larger than the required area, as follows:

$$A_j\left(\underline{x}_j\right) \geq \left(1 + \widehat{a}_{exc,j} / 100\right) \frac{\widehat{Q}_j}{U\left(\underline{x}_j\right) F\left(\underline{x}_j\right) \widehat{LMTD}_j} \tag{2}$$

where $\widehat{Q}_j$ and $\widehat{LMTD}_j$ are the heat load and the logarithmic mean temperature difference in the heat exchanger j, $\widehat{a}_{exc,j}$ is the excess area that is many times required because of uncertainties in the process flowrate and physical properties, $F(\underline{x}_j)$ is the correction factor of the logarithmic mean temperature difference, a function of the inlet and outlet temperatures and the heat exchanger type and configuration, which is associated with the set $\underline{x}_j$. Finally, $U(\underline{x}_j)$ is the overall heat transfer coefficient, also a function of the independent variables, for which there are several models, depending on the type of the heat exchanger (shell-and-tube, plate, air cooler, etc.).

The general optimization problem of the simultaneous design of all heat exchangers present in the network can be formulated as:

$$Min_X \quad TCCOST = \sum_j CCOST_j\left(\underline{x}_j\right) \tag{3}$$

s.t.

$$\underline{g}_j\left(\underline{x}_j\right) \leq 0 \ \forall j \in J \tag{4}$$

$$\sum_{j \in P_k} \Delta P_{j,k}\left(\underline{x}_j\right) \leq \widehat{\Delta Plim}_k \ \forall k \tag{5}$$

where the search space X is the set union of all search spaces of all individual exchangers. In classical optimization models, the variables of this search space are the discrete values of each geometrical variable, which is not the case for the technique used in this article (see below). In turn, $TCCOST$ is the total capital cost and $CCOST_j(\underline{x}_j)$ is the capital cost of exchanger j, usually expressed in terms of the heat transfer area (Smith, 2005), but amenable to be calculated in more detail (Caputo et al., 2016, 2022). Next, $\Delta P_{j,k}(\underline{x}_j)$ is the pressure drop associated with the network stream k through heat exchanger j. Finally, P_k is the path of stream k through the network, which is given by the set of exchangers j through which the stream k passes, and $\widehat{\Delta Plim}_k$ is the limit of pressure drop for it.

In general, the cost of heat exchangers is given by a nonlinear function of area, as follows (Smith, 2005; Caputo et al., 2016, 2022):

$$CCOST\left(\underline{x}_j\right) = \widehat{C}_{EF,nl} + \widehat{C}_{EV,nl} A_j^{\widehat{b}} \tag{6}$$

where $\widehat{C}_{EF,nl}$, $\widehat{C}_{EV,nl}$ and $\widehat{b}$ are the fixed and variable costs and the area exponent, respectively. However, sometimes they are also represented by a linear relationship, as follows (Barbaro and Bagajewicz, 2005):

$$CCOST\left(\underline{x}_j\right) = \widehat{C}_{EF} + \widehat{C}_{EV} A_j \tag{7}$$

Therefore, if one uses the linear relationship, the objective function can be reduced to the sum of the areas of all exchangers.

It is important to observe that the conventional approach to solving this problem in the literature involves establishing individual values of maximum pressure drop for the design of each heat exchanger. However, this approach ignores the opportunity to optimally distribute the total available pressure drop of a stream among the different heat exchangers, which is explored in the present paper.

An alternative more rigorous option, not explored in this article, is to construct a large mixed-integer nonlinear model (MINLOM), solve it using global MINLP procedures (Mizutani et al., 2003), or a reformulated equivalent mixed-integer linear model linear (MILOM) and solve it using mixed-integer linear programming (MILP) procedures (Gonçalves et al., 2019). However, these approaches usually exhibit convergence issues if a nonlinear model is used, or memory requirements limitations, if a reformulated linear model is employed. In both cases, the computational time is large. We choose a different approach.

The proposed approach in this article avoids these obstacles, combining the elimination of infeasible candidates for each heat exchanger using Set Trimming, the further reduction of the number of solution candidates through the elimination of non-optimal alternatives, and the identification of the optimal set of candidates through the

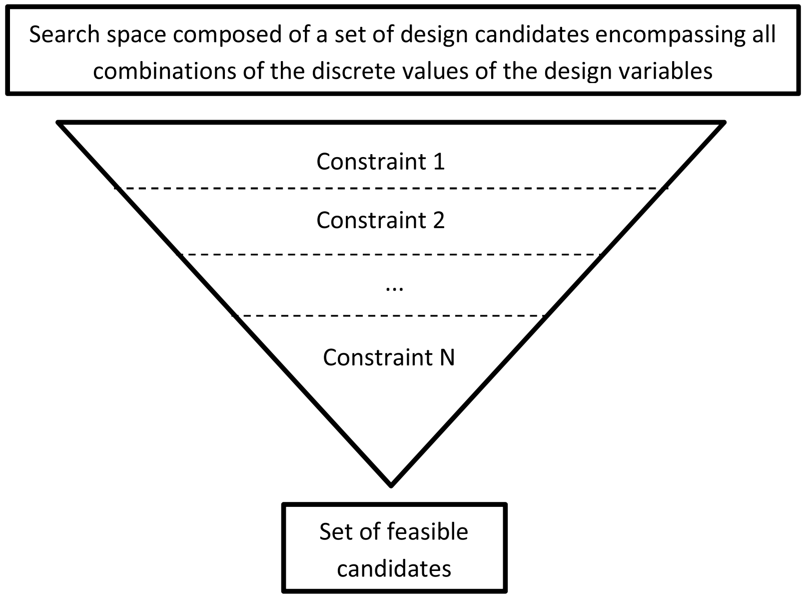

Fig. 1. Complete set trimming algorithm.

solution of an ILP problem.

3. Set trimming used for single exchangers

The search space of the design optimization of a single heat exchanger is composed of a large set of candidates. Each design candidate is composed of discrete values of the design variables. The Set Trimming procedure allows the identification of the feasible set of candidates (Costa and Bagajewicz, 2019), which is a smaller subset of the search space that contains heat exchanger candidates that can fulfill the thermal task.

Set Trimming explores the search space described above against all inequality constraints, one at a time, to eliminate candidates that do not comply with the constraints. Fig. 1 illustrates the gradual elimination of infeasible solution candidates after each trimming during the Set Trimming procedure. At the end of the process, Set Trimming yields the set of feasible candidates (which is considerably smaller than the initial search space). For the design optimization of an individual heat exchanger, the optimal solution is the candidate with the smallest value of the objective function, which can be identified by a simple sorting procedure (Lemos et al., 2020).

Therefore, the application of Set Trimming to a heat exchanger j in a network allows the determination of the set of feasible candidates (FS_j). In the analysis of all heat exchangers in a network, instead of applying the Set Trimming procedure repeatedly, always starting from the same initial set, a proper computational scheme reduces the computational effort, as shown below.

Let PS_j be the initial search space of all exchangers of the same type, called the Primordial Set. The set PS_j is composed of elements i, which are combinations of the discrete values of the geometrical variables, represented by $\underline{x}_{j,i}$. Thus, PS_j is given by:

$$PS_j = \left\{ i, \text{ such that, } \underline{x}_{j,i} \text{ is a candidate for the generic heat exchanger } j \right\} \tag{8}$$

Once the primordial set is constructed, Set Trimming can be used to obtain an Initial Set, which is, in principle, the same for all exchangers of the same type. This initial set is obtained by performing Set Trimming on all geometrical constraints, represented by $g_j^{GEOM}(\underline{x}_{j,i}) \leq 0$ (i.e. constraints that do not depend on the specifications of the heat exchanger task), as follows:

$$IS_j = \left\{ i \in PS_j, \text{ such that, } g_j^{GEOM}\left(\underline{x}_{j,i}\right) \leq 0 \right\} \tag{9}$$

Considering the case of shell-and-tube heat exchangers, examined in this paper in more detail, the set of constraints employed in the trimmings from PS_j to IS_j are listed below.

Bounds on the ratio between the tube length (L) and shell diameter (Ds) (Taborek, 2008a):

$$L \geq 3\,Ds \tag{10}$$

$$L \leq 15\,Ds \tag{11}$$

Bounds on the baffle spacing (lbc) (Taborek, 2008b):

$$lbc \geq 0.2\,Ds \tag{12}$$

$$lbc \leq 1.0\,Ds \tag{13}$$

After obtaining the set IS_j, it can be used for the determination of the feasible sets for all heat exchangers of the same type through the application of the rest of the design (instead of using the larger set PS_j, which reduces the computational effort):

$$FS_j = \left\{ i \in IS_j, \text{ such that, } \underline{x}_{j,i} \text{ is feasible} \right\} \tag{14}$$

The constraints employed in the trimmings for generating FS_j from IS_j for shell-and-tube heat exchangers are listed below.

Bounds on the shell-side and tube-side velocities (vs and vt) (Smith, 2005):

$$vs \geq \widehat{vsmin} \tag{15}$$

$$vs \leq \widehat{vsmax} \tag{16}$$

$$vt \geq \widehat{vtmin} \tag{17}$$

$$vt \leq \widehat{vtmax} \tag{18}$$

Bounds on the Reynolds number in the tube-side and shell-side flows:

$$Ret \geq \widehat{Retmin} \tag{19}$$

$$Ret \leq \widehat{Retmax} \tag{20}$$

$$Res \geq \widehat{Resmin} \tag{21}$$

$$Res \leq \widehat{Resmax} \tag{22}$$

Additionally, the last trimming to generate FS_j is associated with Eq. (2), which expresses the need that the area must be larger than the required area.

4. Reduction of the feasible sets of candidates

Each solution candidate j of the feasible set FS_j is associated with a heat transfer area (and, consequently, a capital cost) and a pair of pressure drops for the hot and cold streams. If the total pressure drop of a stream that passes through several exchangers is limited, the solution candidates of each heat exchanger in this stream that have a higher pressure drop hinder the selection of less expensive candidates for the other heat exchangers through which the same stream goes through. Therefore, solution candidates in the set FS_j that are associated with higher areas (higher capital costs) and higher pressure drops than others can be eliminated because less expensive alternatives with lower pressure drops allow global solutions for the network with smaller values of the objective function. In addition, feasible exchangers that exhibit a pressure drop in one or both streams that is higher than the overall pressure drop assigned to those streams can also be eliminated. These two elimination procedures render a reduced set of feasible candidates (RS_j).

5. Identification of the optimal solution

The Set Trimming procedure described above identified all feasible candidates for each exchanger ($\underline{x}_{j,i}$), then some remaining candidates were eliminated because better options exist among the set of solution alternatives. Now, the best design alternative for each heat exchanger is identified, considering that the sum of the pressure drops of the heat exchangers in the same stream must obey the available pressure drop for that stream. This optimal solution is obtained through a linear optimization problem, therefore, there are no convergence limitations or multiple local optima with different values of the objective function, i.e. the global optimum is always identified. The next paragraphs show the mathematical formulation of the optimization problem.

Associated with each solution candidate, there is a pressure drop through the exchanger ($\widehat{\Delta P}_{j,k}$). Thus, the problem consists now of picking one candidate i, for each heat exchanger j, in such a manner that the sum of the pressure drops along the path of the streams abides by the pressure drop limit of the stream. This is accomplished by solving the following integer-linear optimization model (ILOM), which is one form of the knapsack problem with multiple constraints:

$$Min_{\forall i \in FS_j, j}\ TCCOST = \sum_{j,i} CCOST\left(\underline{x}_{j,i}\right) y_{j,i} \tag{23}$$

s.t.

$$\sum_{i \in FS_j, j \in P_k} \widehat{\Delta P}_{j,k}\left(\underline{x}_{j,i}\right) y_{j,i} \leq \widehat{\Delta Plim}_k\ \forall k \tag{24}$$

$$\sum_{i \in RS_j} y_{i,j} = 1\ \forall j \tag{25}$$

$$y_{j,i} \in \{0,1\}^{K_j}\ \forall i \in RS_j, j \tag{26}$$

where $y_{j,i}$ is a binary variable that indicates that the candidate i (composed of a set of discrete values of the design variables, $\underline{x}_{j,i}$) is selected to the task related to heat exchanger j, and K_j is the sum of cardinalities of the sets RS_j. The model parameter $\widehat{\Delta P}_{j,k}(\underline{x}_{j,i})$ corresponds to the pressure drop of each candidate that is evaluated using a proper fluid dynamic model.

Table 1
Discrete values of the design variables.

Variable	Values
Outer tube diameter, *dte* (m)	0.01905, 0.02540, 0.03175, 0.03810, 0.05080
Tube length, *L* (m)	1.2192 1.524 1.8288 2.1336 2.4384 2.7432 3.048 3.3528 3.6576 3.9624 4.2672 4.572 4.8768 5.1816 5.4864 5.7912 6.096
Number of baffles, *Nb*	1, 2, …, 20
Number of tube passes, *Ntp*	1, 2, 4, 6
Tube pitch ratio, *rp*	1.25, 1.33, 1.50
Shell diameter, *Ds* (m)	0.2032 0.254 0.3048 0.33655 0.38735 0.43815 0.48895 0.53975 0.59055 0.635 0.6858 0.7366 0.7874 0.8382 0.889 0.9398 0.9906 1.0668 1.143 1.2192 1.3716 1.524 1.6764 1.8288 1.9812 2.1336 2.286 2.4384 2.7432 3.048
Tube layout, *lay*	1 (30° layout), 2 (90° layout), 3 (45° layout)
Baffle cut ratio, *Bc*	0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45

The above ILOM can be solved using an integer linear programming (ILP) algorithm (e.g. branch-and-bound).

An alternative approach to the above ILOM is to generate discrete candidates for set X (i.e. each solution candidate would be a set of discrete values of the design variables of all heat exchangers together) and solve the problem using Complete Set Trimming, performing an additional trimming using the constraint represented in Eq. (5), and then proceed to sorting. This option is impractical because the cardinality of the initial set for set trimming for each exchanger is large ($\sim 10^5$) (see Lemos et al., 2020), which results in a too-large initial set for all exchangers, ($\sim 10^{5N}$ elements, where N is the number of exchangers). Another alternative approach would be to apply a Complete Set Trimming on a set of combinations of members of the feasible sets RS_j to generate a set of candidate combinations of exchangers, where the constraint in Eq. (5) is used for trimming (the size of this set is $\sim 10^{3N}$), which is still too large and impractical.

6. Detailed global optimization procedure

The proposed optimization procedure for the simultaneous design of all heat exchangers in a network can be employed for problems involving any network structure, including stream splits and loops. The optimality of the network structure does not affect the application of the proposed procedure, i.e. it can be employed for optimal or nonoptimal network structures. The procedure is also versatile in relation to the thermofluid dynamic model of the heat exchangers, i.e. it can be applied to any heat exchanger model for evaluation of the required area and pressure drops.

Without loss of generality, the complete optimization procedure is represented here for heat exchanger networks composed of shell-and-tube heat exchangers:

1- The Primordial Search Space (PS), of all combinations of design parameters is generated for all heat exchangers of the network (because there are only shell-and-tube heat exchangers in the network, PS is unique for all heat exchangers).
2- Geometrical constraints (Eqs. (10–13)) are applied to PS to obtain the initial set (IS).
3- The rest of the design constraints (Eqs. (15–22) and Eq. (2)) are applied to IS to obtain the feasible set for each heat exchanger j, FS_j.
4- The pressure drops in the tube-side and shell-side of all feasible candidates are evaluated.
5- An analysis of the heat transfer area and pressure drops involving each solution candidate of each heat exchanger is conducted to generate the sets RS_j (Eq. (23)).

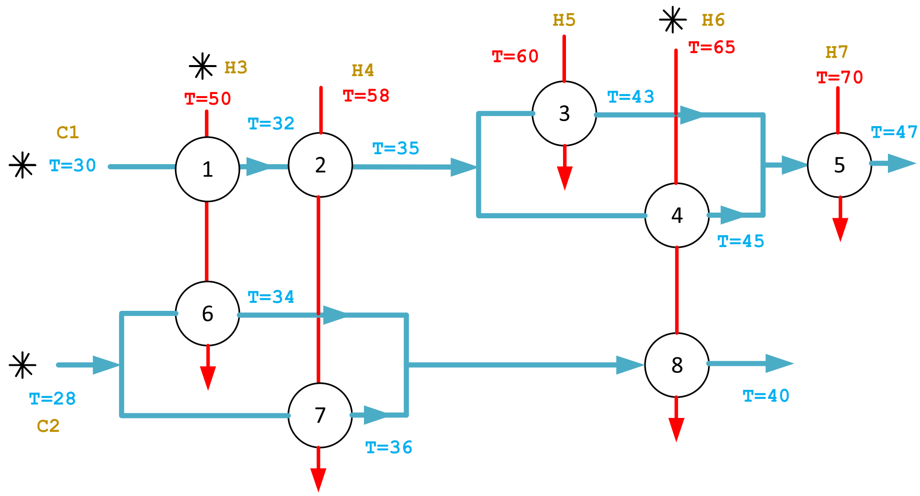

Fig. 2. Example 1: Schematic diagram of the HEN.

Table 2
Example 1: Mass flow rate and thermo-physical properties of the streams.

Stream	Mass flow rate (kg/s)	Viscosity (Pa·s)	Thermal conductivity (W/(m·K))	Density (kg/m^3)	Heat capacity (J/(kg·K))
C1	200	$\mu(T)$	0.145	888.1	1881
C2	220	0.001	0.6	999.4	4186
H3	150	0.00339	0.13	840	1930
H4	140	0.004	0.115	890	2220
H5	150	0.004	0.115	890	2220
H6	180	0.004	0.115	890	2220
H7	100	0.004	0.115	890	2220

6- The ILOM problem depicted in Eqs. (24–27) is solved using an ILP procedure to identify the optimal design of each heat exchanger, complying with the pressure drop constraints of the network streams.

7. Results and discussion

Two examples are explored to illustrate the application of the proposed design procedure. The network examples are composed of shell-and-tube heat exchangers with a single E-type shell. The search space is composed of the geometrical options depicted in Table 1. With this option, the search space has 12,852,000 different candidates for each heat exchanger in the network. This search space for each heat exchanger is considerably larger than the search spaces used before. For example, the search space used by Lemos et al. (2020) has 168,000 candidates. The following parameters are used for the nonlinear capital cost: $\widehat{C}_{EF,nl} = 8500$, $\widehat{C}_{EV,nl} = 409$ and $\widehat{b} = 0.85$.

The minimum and maximum flow velocities on the shell-side are 0.2 m/s and 3 m/s (Eqs. (15–16)), and on the tube-side are 0.5 m/s and 3 m/s (Eqs. (17–18)). The minimum and maximum values for the Reynolds number for the tube-side flow are 2500 and 124,000, and the corresponding bounds for the shell-side flow are 0 and 100,000.

All test runs were executed using a computer with a processor i7-4870HQ 2.5 GHz with 16 GB of RAM memory. The computational routines were implemented in Matlab 2023b The MILP code used to solve the ILP problem was the *intlinprog* function available in Matlab. The thermal and pressure drop models employed in the Set Trimming procedure are the same employed by Sanaye and Hajabdollahi (2010): the Bell-Delaware method for the shell-side and the Dittus-Boelter correlation for the tube-side, except for the condenser mentioned in Example 2, which is based on the model employed by Hajabdollahi et al. (2011).

Example 1. Consider the HEN shown in Fig. 1. There are eight heat exchangers with two main cold streams and five hot streams with the given supply and target temperatures. The stars (*) in Fig. 2 identify the streams that are limited by pressure drop (C1, C2, H3, and H6). The heat exchangers in the remaining streams (H4, H5, and H7) are optimized considering individual pressure drop constraints.

The inlet temperatures of heat exchangers 6, 7, and 8 are determined using energy balances. The mass flow rates of the network streams are depicted in Table 2. The split values in the streams C1 and C2 are set at 0.6 and 0.3, respectively.

The thermo-physical properties of the streams are also listed in Table 2. Without loss of generality, the fouling factor is neglected and the physical properties of the streams are assumed uniform, except for the viscosity of stream C1, which is evaluated by:

$$\mu(T) = -8.986\ 10^{-10}\ T^5 + 4.202\ 10^{-7}\ T^4 - 7.432{\cdot}10^{-5}\ T^3 + 6.18110^{-3}\ T^2 - 0.2424\ T + 3.71 \tag{27}$$

The viscosity values of the stream C1 used in the evaluation of the heat transfer coefficients of the corresponding heat exchangers are determined using Eq. (27) with the average of the inlet and outlet temperatures.

The set of pressure drop limiting constraints corresponding to Eq. (5) is given by:

$$\Delta P_{ex_1,C1} + \Delta P_{ex_2,C1} + \Delta P_{ex_3,C1} + \Delta P_{ex_5,C1} \le \widehat{\Delta Plim}_{C1} = 350 \text{ kPa} \tag{28}$$

$$\Delta P_{ex_1,C1} + \Delta P_{ex_2,C1} + \Delta P_{ex_4,C1} + \Delta P_{ex_5,C1} \le \widehat{\Delta Plim}_{C1} = 350 \text{ kPa} \tag{29}$$

$$\Delta P_{ex_6,C2} + \Delta P_{ex_8,C2} \le \widehat{\Delta Plim}_{C2} = 210 \text{ kPa} \tag{30}$$

Table 3
Example 1: Cardinalities of the feasible and reduced sets.

Heat exchanger	Feasible set cardinality (FS_j)	Reduced feasible set cardinality (RS_j)
1	431,482	195
2	294,393	178
3	3862	40
4	100,774	166
5	111,659	114
6	280,150	291
7	23,769	129
8	20,933	90
Total	1267,022	1203

Table 4
Example 1: Optimal values of area and pressure drops for area minimization.

Heat exchanger	A (m^2)	ΔP_c (kPa)	ΔpH (kPa)
1	177.31	63.948	2.541
2	239.91	57.274	14.241
3	460.78	148.390	1.7459
4	169.76	138.798	16.997
5	183.68	80.001	41.161
6	70.99	154.197	28.169
7	559.28	59.504	97.084
8	466.25	3.374	123.219
Total	2327.97	705.488	325.161

Table 5
Example 1: Pressure drops of the streams.

Pressure drop constraint	Optimal value (kPa)	Maximum pressure drop (kPa)
$\Delta P_{ex_1,C1} + \Delta P_{ex_2,C1} + \Delta P_{ex_3,C1} + \Delta P_{ex_5,C1}$	349.61	350
$\Delta P_{ex_1,C1} + \Delta P_{ex_2,C1} + \Delta P_{ex_4,C1} + \Delta P_{ex_5,C1}$	340.02	350
$\Delta P_{ex_6,C2} + \Delta P_{ex_8,C2}$	157.57	210
$\Delta P_{ex_7,C2} + \Delta P_{ex_8,C2}$	62.88	210
$\Delta P_{ex_1,H3} + \Delta P_{ex_6,H3}$	30.71	150
$\Delta P_{ex_4,H6} + \Delta P_{ex_8,H6}$	140.22	150
$\Delta P_{ex_2,H4}$	14.24	100
$\Delta P_{ex_3,H5}$	1.75	100
$\Delta P_{ex_5,H7}$	41.16	100
$\Delta P_{ex_7,H4}$	97.08	100

$$\Delta P_{ex_7,C2} + \Delta P_{ex_8,C2} \le \widehat{\Delta Plim}_{C2} = 210 \text{ kPa} \tag{31}$$

$$\Delta P_{ex_1,H3} + \Delta P_{ex_6,H3} \le \widehat{\Delta Plim}_{H3} = 150 \text{ kPa} \tag{32}$$

$$\Delta P_{ex_4,H6} + \Delta P_{ex_8,H6} \le \widehat{\Delta Plim}_{H6} = 150 \text{ kPa} \tag{33}$$

$$\Delta P_{ex_2,H4},\ \Delta P_{ex_3,H5}, \Delta P_{ex_5,H7}, \Delta P_{ex_7,H4} \le 100 \text{ kPa} \tag{34}$$

The cardinality of the Primordial and Initial Sets is 12,852,000 and 4725,000, respectively. Table 3 shows the cardinalities of the Feasible Sets (FS_j) and the Reduced Feasible Sets (RS_j) for each heat exchanger in the network. The total number of solution candidates in the sets FS_j and RS_j are 1267,022 and 1203, respectively. These values are associated with the number of binary variables in the ILP problem (Eqs. (24–27)), which indicates a reduction of the dimension of the problem of 99.88 % through the procedure of reduction of the number of feasible candidates (i.e. the transition from FS_j to RS_j).

The optimal result using the total area as an objective function reveals that the minimum heat transfer surface area is 2328 m^2 (Table 4). Table 5 compares the available pressure drops for the streams with the results of the optimization, where one notices that the pressure drop for the first three streams resulted in values close to a maximum. The optimal values of the design variables of each heat exchanger are shown in Table 6.

The computational times employed in each step of the algorithm are shown in Table 7, encompassing Set Trimming, reduction of the feasible set, and the solution of the ILP problem. For comparison, Table 7 also shows the computational time of a run without the reduction of the feasible set. The reduction of the feasible set brings a reduction of 59.52 % in the computational time. Particularly, the time consumed to solve the ILP is reduced by 99.97 %.

Table 7
Example 1: Computational times (seconds).

Method	Set Trimming (ST)	Reduction of the feasible set (RED)	Integer Linear Programming (ILP)	Total
ST+ ILP	854.6	0	1269	2123.6
ST+ RED +ILP	854.6	4.64	0.39	859.63

We now compare with a case where no constraints are added on pressure drop for the streams. Tables 8 and 9 show the areas and pressure drops of the system when no constraints of pressure drop are added. These were obtained by picking the candidate of a smaller area for each exchanger (if there is more than one heat exchanger with the smallest area, the alternative with the smaller pressure drop was selected). In this case, there is a reduction of the total area of 32.21 %, with a considerable increase in the streams' pressure drop, e.g. the pressure drop for stream C1 becomes more than six times higher than the original maximum values for this stream.

We also compare with the results of design optimization of heat exchangers using the minimization of the total annualized cost, TAC (interest rate of 7 %, and a time horizon of 20 years). In this case, there are no available pressure drops imposed on the exchangers, and the total pressure drop along the heat exchangers is associated with pumping costs (energy price equal to 0.08 $/kWh), which are included in the objective function.

The optimal results show a minimum cost of 105,272 $/year and a corresponding total heat transfer surface area of 4275.32 m^2 (Table 10). The total area is larger and the pressure drops are smaller than the corresponding results based on the maximum pressure drop for each stream.

Example 2. As the second example, consider the HEN shown in Fig. 3. This HEN represents a typical crude preheat train present in petroleum refineries. The objective of this example is to show that the proposed procedure can be applied to large-size problems. There are 23 heat exchangers with the main cold stream being crude oil. The crude oil with a mass flow rate of 64.81 kg/s enters the system at 30 °C and is heated to 233.4 °C by the last heat exchanger (heat exchanger 15). A fired heater further increases the temperature to 350 °C. The hot stream fluids, as well as the mainstream cooling, are then cooled using cooling water (CW) in heat exchangers 16–23. Mass flow rates, inlet temperatures, and thermo-physical properties of the hot streams in each exchanger are listed in Table 11. Note that the hot stream in the first heat exchanger is overheated naphtha and is condensed. As a result, only the first heat exchanger is a shell-and-tube condenser (in this case, the properties depicted in Table 11 correspond to the condensate). Finally, the mass flow rates of the crude oil in heat exchangers 3, 8, and 10 are, 47.45 kg/

Table 6
Example 1: Optimal values of the design variables for each heat exchanger.

Heat exchanger	do (m)	L (m)	Nb	Npt	rp	Ds (m)	lay (°)	Bc	Cold side allocation
1	0.0254	4.572	4	1	1.5	0.9906	30	0.15	Tube side
2	0.0254	5.4864	5	2	1.5	1.0668	30	0.15	Tube side
3	0.01905	6.096	8	1	1.25	0.9906	30	0.15	Tube side
4	0.01905	5.4864	10	1	1.25	0.6858	45	0.15	Tube side
5	0.01905	5.791	5	2	1.5	0.7874	30	0.15	Tube side
6	0.01905	4.2672	16	1	1.33	0.5397	90	0.15	Shell side
7	0.01905	6.096	12	4	1.25	1.2192	90	0.15	Shell side
8	0.01905	6.096	18	1	1.25	1.0668	45	0.15	Shell side

Table 8
Example 1: Areas of constrained and unconstrained solutions.

Heat exchanger	Area for the pressure-constrained solution (m^2)	Area for the unconstrained pressure solution (m^2)
1	177.31	76.07
2	239.91	84.06
3	460.78	310.83
4	169.75	131.35
5	183.68	93.40
6	70.99	70.99
7	559.28	360.09
8	466.25	451.29
Total	2327.97	1578.08

Table 9
Example 1: Pressure drops corresponding to the unconstrained solution.

Pressure drop constraint	Constrained solution (kPa)	Unconstrained solution (kPa)
$\Delta P_{ex_1,C1} + \Delta P_{ex_2,C1} + \Delta P_{ex_3,C1} + \Delta P_{ex_5,C1}$	349.61	2227.51
$\Delta P_{ex_1,C1} + \Delta P_{ex_2,C1} + \Delta P_{ex_4,C1} + \Delta P_{ex_5,C1}$	340.02	2313.35
$\Delta P_{ex_6,C2} + \Delta P_{ex_8,C2}$	157.571	178.22
$\Delta P_{ex_7,C2} + \Delta P_{ex_8,C2}$	62.88	26.86
$\Delta P_{ex_1,H3} + \Delta P_{ex_6,H3}$	30.71	58.35
$\Delta P_{ex_4,H6} + \Delta P_{ex_8,H6}$	140.22	167.68

s, 45.14 kg/s, and 50.93 kg/s, respectively.

The thermo-physical properties of crude oil are considered as a function of temperature, as shown in the following equations (the evaluation of the heat transfer coefficients in the heat exchangers is based on physical properties evaluated at the average of the inlet and outlet temperatures):

$$\rho(T) = 0.0018T^2 - 1.0921T + 872.9021 \tag{35}$$

$$Cp(T) = 0.39325T^3 - 0.0206T^2 + 6.8665T + 1630.1 \tag{36}$$

$$k(T) = -0.0001T + 0.1006 \tag{37}$$

The evaluation of the crude oil viscosity employs Eq. (27).

We consider that the total pressure drop of some streams is subject to a maximum pressure drop $\widehat{\Delta P}_{p,\ max,\ i} = \widehat{N}_{p,i}\ \widehat{\Delta P}_{max,\ exch}$, where $\widehat{N}_{p,i}$ and $\widehat{\Delta P}_{max,\ exch}$ are the number of exchangers allocated for stream i in path p, and the maximum pressure drop per exchanger, respectively. Thus, for this example, we write the pressure drop constraints as follows:

$$\Delta P_{ex_1,C} + \Delta P_{ex_2,C} + \Delta P_{ex_5,C} + \Delta P_{ex_6,C} + \Delta P_{ex_7,C} \le 5\ \widehat{\Delta P}_{max,\ exch} \tag{38}$$

$$\Delta P_{ex_1,C} + \Delta P_{ex_2,C} + \Delta P_{ex_3,C} + \Delta P_{ex_4,C} + \Delta P_{ex_7,C} \le 5\ \widehat{\Delta P}_{max,\ exch} \tag{39}$$

$$\Delta P_{ex_1,C} + \Delta P_{ex_2,C} + \Delta P_{ex_5,C} + \Delta P_{ex_6,C} + \Delta P_{ex_8,C} \le 5\ \widehat{\Delta P}_{max,\ exch} \tag{40}$$

Table 10
Example 1: Optimal values of area and pressure drops for TAC minimization.

Heat Exchanger	Area (m^2)	ΔP_c (kPa)	**ΔpH** (kPa)
1	417.97	18.14	1.82
2	506.00	22.11	2.16
3	763.95	55.26	8.48
4	390.80	14.33	3.05
5	353.50	31.49	1.36
6	165.92	5.86	4.64
7	838.59	3.30	15.68
8	838.59	6.08	10.12
Total	4275.32	156.57	47.31

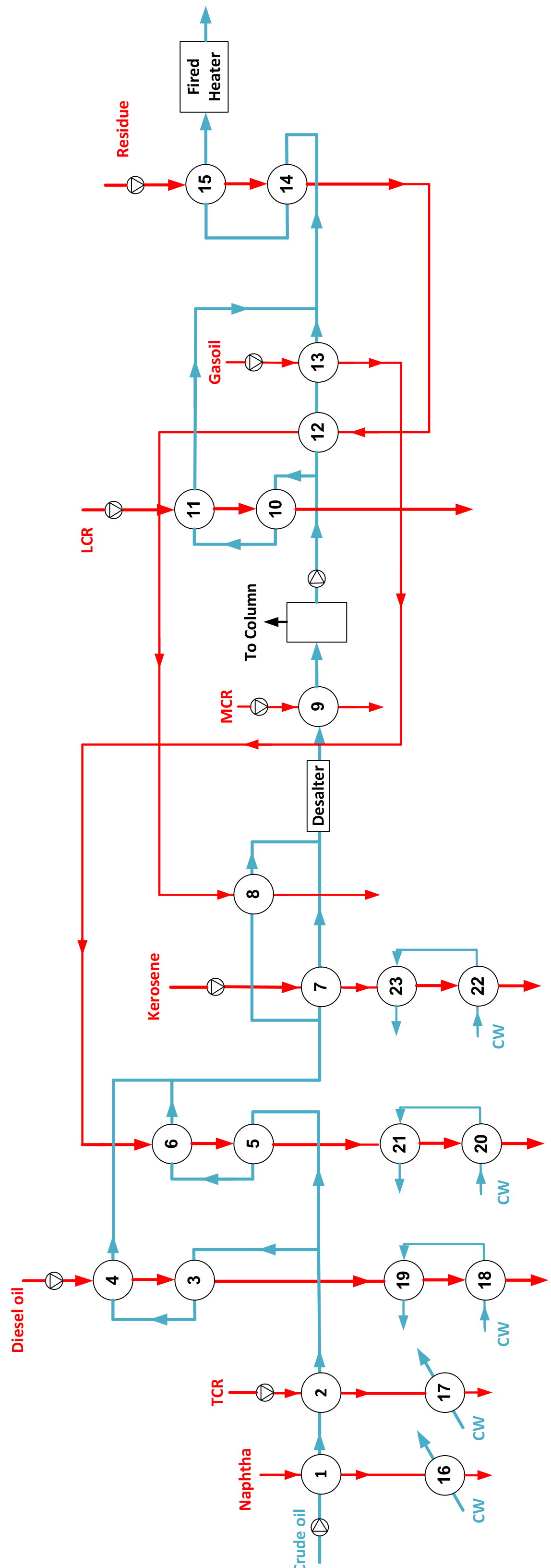

Fig. 3. Schematic diagram of the HEN for the second example.

Table 11
Example 2: Stream data and thermo-physical properties of the hot streams.

Heat exchanger	Mass flow rate (kg/s)	Inlet Temperature (°C)	Viscosity (Pa·s)	Thermal conductivity (W/(m·K))	Density (kg/m3)	Heat capacity (J/(kg·K))	Fouling factor (m^2·K/W)
1	10.18	125	0.00010	0.09515	725.747	2093.40	0.00018
2	245.36	134	0.00024	0.07785	689.459	2395.78	0.00018
3	17.36	146.15	0.00085	0.07785	784.714	2279.48	0.00009
4	17.36	260	0.00085	0.07785	784.714	2279.48	0.00009
5	9.24	147.9	0.00040	0.06920	734.819	2232.96	0.00009
6	9.24	238	0.00040	0.06920	734.819	2232.96	0.00009
7	6.36	185	0.00031	0.06920	716.675	2279.48	0.00009
8	13.3	205	0.0048	0.07439	843.681	3314.55	0.00018
9	75.22	225	0.00027	0.06747	721.211	2628.38	0.00009
10	87.5	227.5	0.00025	0.06055	675.852	2674.90	0.00009
11	87.5	270	0.00025	0.06055	675.852	2674.90	0.00009
12	13.3	240	0.0048	0.07612	843.681	2500.45	0.00018
13	17.36	280	0.00046	0.06920	743.890	2535.34	0.00009
14	13.3	254.08	0.00163	0.06920	802.857	2826.09	0.00018
15	13.3	335	0.00163	0.06920	802.857	2826.09	0.00018
16	23.36	40	0.00035	0.10553	721.211	2028.27	0.00018
17	245.36	115	0.00038	0.08650	725.747	1958.02	0.00018
18	9.24	71.53	0.0030	0.09688	855.020	2128.29	0.00009
19	9.24	122	0.0030	0.09688	855.020	2128.29	0.00009
20	17.36	64.56	0.019	0.09515	889.040	2128.29	0.00009
21	17.36	122	0.019	0.09515	889.040	2128.29	0.00009
22	6.36	74.83	0.0018	0.09342	817.826	2128.29	0.00009
23	6.36	148	0.0018	0.09342	817.826	2128.29	0.00009

Table 12
Example 2: Optimal area and pressure drops for area minimization.

Heat Exchanger	A (m^2)	ΔP_c (kPa)	**ΔpH** (kPa)
1	8.573	22.45	6.58
2	94.223	2.75	12.35
3	189.577	38.66	3.39
4	247.455	71.51	2.43
5	710.687	78.10	5.69
6	531.3	36.56	5.43
7	41.882	28.49	31.71
8	173.287	31.63	11.17
9	47.281	160.61	16.36
10	12.477	75.30	44.16
11	1.9457	12.67	1.95
12	18.606	24.19	4.64
13	29.916	92.37	7.91
14	131.3522	12.52	10.89
15	152.839	9.55	8.14
16	3.405	8.08	35.20
17	928.563	16.82	52.15
18	13.899	41.80	47.84
19	18.022	376.68	83.41
20	93.396	13.11	28.09
21	1.1456	58.76	21.78
22	15.961	276.23	36.59
23	15.517	155.73	35.35
Total	3580.319	1644.65	513.32

$$\Delta P_{ex_1,C} + \Delta P_{ex_2,C} + \Delta P_{ex_3,C} + \Delta P_{ex_4,C} + \Delta P_{ex_8,C} \le 5\ \widehat{\Delta P}_{max,\ exch} \tag{41}$$

$$\Delta P_{ex_{12},C} + \Delta P_{ex_{13},C} + \Delta P_{ex_{14},C} + \Delta P_{ex_{15},C} \le 4\ \widehat{\Delta P}_{max,\ exch} \tag{42}$$

$$\Delta P_{ex_{10},C} + \Delta P_{ex_{11},C} + \Delta P_{ex_{14},C} + \Delta P_{ex_{15},C} \le 4\ \widehat{\Delta P}_{max,\ exch} \tag{43}$$

$$\Delta P_{ex_2,TCR} + \Delta P_{ex_{17},TCR} \le 2\ \widehat{\Delta P}_{max,\ exch} \tag{44}$$

$$\Delta P_{ex_3,Diesel} + \Delta P_{ex_4,Diesel} + \Delta P_{ex_{18},Diesel} + \Delta P_{ex_{19},Diesel} \le 4\ \widehat{\Delta P}_{max,\ exch} \tag{45}$$

$$\Delta P_{ex_5,GSL} + \Delta P_{ex_6,GSL} + \Delta P_{ex_{13},GSL} + \Delta P_{ex_{20},GSL} + \Delta P_{ex_{21},GSL} \le 5\ \widehat{\Delta P}_{max,\ exch} \tag{46}$$

$$\Delta P_{ex_7,Kerosene} + \Delta P_{ex_{22},Kerosene} + \Delta P_{ex_{23},Kerosene} \le 3\ \widehat{\Delta P}_{max,\ exch} \tag{47}$$

$$\Delta P_{ex_8,Residue} + \Delta P_{ex_{12},Residue} + \Delta P_{ex_{14},Residue} + \Delta P_{ex_{15},Residue} \le 4\ \widehat{\Delta P}_{max,\ exch} \tag{48}$$

$$\Delta P_{ex_9,MCR} \le \widehat{\Delta P}_{max,\ exch} \tag{49}$$

$$\Delta P_{ex_{10},LCR} + \Delta P_{ex_{11},LCR} \le 2\ \widehat{\Delta P}_{max,\ exch} \tag{50}$$

The pressure drop of the crude oil stream in heat exchanger 9 and the pressure drop in the cooling water streams are not constrained.

Assuming $\widehat{\Delta P}_{max,\ exch} = 35$ kPa, the optimal results of each heat exchanger surface area as well as pressure drops are listed in Table 12. The total heat transfer surface area obtained is 3580.319 m^2. In addition, the optimal values of the design variables for each heat exchanger are listed in Table 13.

The computational times of each step to solve Example 2 are depicted in Table 14. Note that, despite the large size of the problem, the total computational time is reasonable for practical applications in process design. In this case, Set Trimming is the most time-consuming step, due to the large number of heat exchangers present in the network, but the total computational time is <40 min, which shows that the application of the proposed approach for large-size problems is viable. Considering that each of the exchangers' trimming step could be done in parallel without any loss in rigor, the total time can be reduced to about 1.6 min for Example 2 this estimate was obtained by dividing the time needed for set trimming (2236 s) by 23, the number of exchangers.

Aiming at comparing the result of the proposed approach with the design practice, where each heat exchanger is subject to individual pressure drop limits, we solve the problem considering a maximum pressure drop of $\widehat{\Delta P}_{max,\ exch}$= 35 kPa, applied to each heat exchanger associated with Eqs. (38–50) (i.e. the total maximum pressure drop of each hydraulic constrained stream was distributed uniformly for the design of each heat exchanger). The corresponding results are depicted in Table 15. The increase in area compared with the optimal solution is 9.8 %. The usage of larger $\widehat{\Delta P}_{max,\ exch}$ values render smaller differences. Considering that Example 2 is based on a typical crude preheat train, this comparison shows the potential economic gain that can be attained using the proposed approach for the design of real systems. Indeed, Table 12 is the result of limiting the total pressure drop for some streams and let the optimization approach establishes how the pressure drop

Table 13
Example 2: Optimal values of the design variables for each heat exchanger.

Heat Exchanger	do (m)	L (m)	Nb	Npt	rp	Ds (m)	lay (°)	Bc	Cold side allocation
1	0.0381	1.524	1	4	1.33	0.48895	90	0.150	Tube side
2	0.0508	3.9624	3	2	1.50	1.2192	90	0.150	Tube side
3	0.01905	5.791	20	4	1.25	0.7366	45	0.150	Tube side
4	0.01905	5.791	19	6	1.25	0.8382	45	0.150	Tube side
5	0.01905	6.096	18	6	1.25	1.3716	45	0.150	Tube side
6	0.01905	5.791	16	4	1.25	1.2192	45	0.150	Tube side
7	0.01905	4.2672	7	6	1.33	0.43815	90	0.150	Shell side
8	0.01905	5.791	20	2	1.25	0.6858	45	0.150	Tube side
9	0.01905	2.4384	5	2	1.33	0.59055	90	0.150	Shell side
10	0.0254	2.7432	10	4	1.25	0.33655	90	0.150	Shell side
11	0.0508	1.2192	2	2	1.33	0.3048	30	0.150	Shell side
12	0.0381	1.8288	2	2	1.25	0.59055	90	0.150	Shell side
13	0.0254	1.8288	4	2	1.33	0.59055	30	0.150	Shell side
14	0.01905	5.791	20	1	1.25	0.59055	45	0.150	Tube side
15	0.01905	5.791	19	1	1.25	0.635	45	0.150	Tube side
16	0.0254	1.524	4	2	1.33	0.254	90	0.150	Tube side
17	0.01905	5.4864	7	1	1.33	1.6764	90	0.150	Tube side
18	0.01905	1.8288	4	6	1.33	0.38735	90	0.150	Shell side
19	0.01905	3.9624	17	4	1.33	0.3048	90	0.150	Shell side
20	0.01905	6.096	20	1	1.25	0.48895	45	0.150	Tube side
21	0.01905	5.4864	18	2	1.25	0.53975	45	0.150	Tube side
22	0.03175	4.572	20	6	1.33	0.33655	30	0.150	Shell side
23	0.0254	3.3528	12	6	1.33	0.33655	30	0.150	Shell side

Table 14
Example 2: Computational times (seconds).

Method	Set trimming (ST)	Reduction of the feasible set (RED)	Integer linear programming (ILP)	Total
ST+ ILP	2236	–	198	2434
ST+ RED +ILP	2236	30.04	1.51	2267

Table 15
Example 2: Optimal area and pressure drops for equal pressure drop allocated to each exchanger.

Heat Exchanger	A (m^2)	ΔP_c (kPa)	**ΔpH** (kPa)
1	6.029	33.72	5.49
2	40.715	11.14	20.09
3	217.650	30.78	2.47
4	321.579	33.75	2.99
5	907.367	33.22	6.12
6	578.875	27.45	10.43
7	41.372	18.38	32.40
8	173.288	31.63	11.17
9	47.282	160.61	16.36
10	14.666	11.80	11.25
11	1.946	12.67	1.95
12	18.290	34.39	4.16
13	33.808	33.22	6.56
14	131.352	12.52	10.89
15	152.840	9.55	8.14
16	3.405	8.08	35.20
17	967.163	20.94	31.87
18	16.052	129.91	32.31
19	29.112	155.66	27.77
20	93.396	13.11	28.09
21	100.146	58.76	21.78
22	17.329	151.33	31.68
23	16.588	97.72	31.46
Total	3930.241	1130.45	390.75

must be distributed among the individual heat exchangers of those streams. Thus, some exchangers have a pressure drop higher than the 35 kPa limit. For the cold streams, those are the heat exchangers 3, 4, 5, 6, 10 and 13. These exchangers exhibit collectively 182.5 kPa of extra pressure drop in the cold streams, still compatible with the total pressure drop of each stream, because other heat exchangers present a lower pressure drop. By allowing a higher pressure drop in these heat exchangers, it is possible to reduce the total area of the network. The higher total area associated with the design practice (Table 15) indicates that the practice of decomposing a system to design its individual parts, without taking into account their interactions, yields nonoptimal solutions. It should be also noted that the total pressure drop associated with the solution obtained using the proposed approach is higher than the design practice result. This difference occurs because in the design practice case, if a heat exchanger is associated with a smaller pressure drop than the limit, this difference cannot be employed by other heat exchangers of the same stream. Therefore, the application of pressure drop limits to individual heat exchangers instead of applying the limit to the whole stream yields a solution with a smaller total pressure drop.

8. Conclusions

This study addressed the simultaneous design of heat exchangers that are part of a defined heat exchanger network (HEN). The proposed procedure is versatile, it can be employed to design the thermal equipment in networks with any structure, optimal or nonoptimal; and using any heat exchanger model.

The total heat transfer area (or total capital cost) is optimized while adhering to pressure drop constraints. The design variables for each heat exchanger are: shell diameter, outer tube diameter, number of tube passes, tube pitch ratio, tube layout, tube length, number of baffles, and baffle cut ratio. To achieve a globally optimal solution, Set Trimming was employed in conjunction with an innovative optimization scheme based on constructing an integer linear model. Set Trimming was already employed successfully in the literature to design individual equipment (Lemos et al., 2020), but it is employed here in a different context, limiting the search space for a subsequent mathematical programming problem to solve the design problem of a set of equipment.

The methodology was applied to two case studies, consisting of HENs with 8 and 23 heat exchangers, where each exchanger had 12,852,000 initial design candidates. The computational times for these optimizations were reasonable (under one hour for the larger example). These findings highlight the effectiveness of obtaining globally optimal solutions (as opposed to other approaches) and the computational efficiency of the proposed approach for large-scale HEN design.

The proposed methodology to design all heat exchangers in a network simultaneously, according to the streams' available pressure drops, can be an important tool for future developments in HEN synthesis, particularly for algorithms based on the enumeration of HEN structures.

The design procedure can be improved in the future to address flexibility issues, where the design tasks are not defined by a single set of parameters, because the thermal equipment must be able to operate according to certain bounds.

CRediT authorship contribution statement

Hassan Hajabdollahi: Writing – review & editing, Writing – original draft, Software, Methodology, Formal analysis, Conceptualization. **André L.H. Costa:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis. **Miguel J. Bagajewicz:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

There is no conflict of interest in the preparation and submission of this article

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Data availability

No data was used for the research described in the article.

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